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Research Article

Utilization of Number Line Media in Dienes' Step Learning: A Process Study to Overcome Difficulties in Integer Operations

Abdur Rohim[1]*, Cholis, Sa'dijah[2], Rustanto Rahardi[2], Sisworo[2], Sukoriyanto[2] and Makbul Muksar[2]

[1] Department of Mathematics Education, Universitas Islam Darul 'ulum Lamongan, Jawa Timur, Indonesia
[2] Department of Mathematics Education, Universitas Negeri Malang, Jawa Timur, Indonesia
*Corresponding Author: rohim@unisda.ac.id | Phone Number: +6285649234656

ABSTRACT

Integers are an important topic in mathematics that students must understand well. However, in practice, many students face difficulties in understanding integer operations. One way to address this issue is by using a number line manipulative that employs Dienes' steps. This research is qualitative, with a case study approach. The subjects consisted of three 6th-grade students selected through purposive sampling. The three subjects had varying mathematical abilities: high, medium, and low. The learning activities used six steps of Dienes. The instruments used were observation sheets, interview guidelines, and a test consisting of eight questions related to the addition and subtraction of integers. Data collection techniques followed the Miles and Huberman model, which includes data reduction, data presentation, and drawing conclusions. The results of the study indicate that learning using Dienes' steps with integer manipulatives successfully addressed students' difficulties in performing integer operations, especially in problems involving operations with two adjacent signs.

Keywords: Dienes Steps; Integers; Number Line

1. INTRODUCTION

Mathematics is a crucial subject because it helps students understand and analyze the world around them. Through the description of quantities, shapes, spaces, and patterns, mathematics facilitates the systematic organization of knowledge. Additionally, mathematics plays a significant role in modern life. Galante (2014) states that mathematics is a universal science that forms the foundation of technological development and plays an essential role in various fields, as well as stimulating human thinking (Maulyda, 2020). In this context, integers are one of the fundamental topics that students must master to understand more advanced mathematical concepts.

Integers are an essential part of mathematics that students need to understand well. The objective of teaching integers is to enable students to use this concept to solve problems relevant to daily life. A strong understanding of integers provides valuable knowledge for students in the future (Ramlah, 2024). However, in practice, many students face difficulties in understanding integer operations, particularly with integer subtraction, which often involves complex word problems.

Ananda (2017) observed at SDN 016 Bangkinang Kota that the limited use of manipulatives caused students to struggle with understanding the concept of integer operations, especially in subtracting whole numbers. Furthermore, the teacher's limited understanding of how to effectively use manipulatives worsened students' comprehension of the topic. Pre-tests conducted to assess students' understanding of whole number subtraction showed disappointing results. The average score was low, with many students struggling to complete the problems. This finding is supported by other research, such as Nurjannah et al. (2019), Sidik & Wakih (2019), Benge et al. (2021), Mandasari & Rosalina (2021), Sidik et al. (2021), and Nengsih & Pujiastutik (2021), which found that students still had difficulties in performing integer operations (addition and subtraction).

This suggests that a more effective approach is needed in teaching integer operations, especially subtraction. Therefore, the researcher believes it is important to utilize Dienes' steps through manipulatives to improve students' understanding of this topic. The use of manipulatives is expected to make learning more engaging and help students better understand mathematical concepts. Dienes (1971) suggested that the principle of perceptual variability is vital in mathematics education. This principle can be implemented by using various representations of the material to help students better understand the concepts being taught. Teaching that integrates various representations, including visual, concrete, and

symbolic, can help students achieve a deeper understanding of the concepts. Furthermore, this approach aligns with constructivism, which asserts that learning must involve students actively in the process of understanding, rather than passively transmitting knowledge (Liptak & Scholtzova, 2021).

The approach based on using concrete representations, such as Dienes blocks, supports a more active and enjoyable learning experience for students. This method teaches that mathematical concepts should be introduced through the concrete stage first, followed by the representational and abstract stages, as suggested by Piaget (1973). By using Dienes' steps with number line manipulatives, students can directly observe how integer operations, such as addition and subtraction, work. Students' difficulties in understanding mathematical material can also be addressed by utilizing technology in learning media. Research by Ilmiyah et al. (2024) shows that learning media can enhance students' conceptual understanding. Rohim & Prayogi (2023) added that students' logical thinking skills play an important role in solving algebraic problems, where difficulties often arise due to limited conceptual understanding.

This study aims to describe the process of teaching integer operations using number line media through Dienes' steps. By using appropriate media and an active learning approach, it is hoped that students will better understand the concept of integer operations and improve their learning outcomes. Previous research also supports that the application of active learning strategies can significantly enhance students' conceptual understanding (Rohmah et al., 2024). It is expected that the results of this study will contribute to the development of more effective and engaging mathematics teaching methods for elementary school students (Ramlah, 2024).

2. RESEARCH METHOD

This study applies a qualitative method, which is descriptive in nature and focuses more on analysis. In qualitative research, the process and understanding from the subject's perspective are the main focus. The approach used is a case study, which aims to explore in depth the use of Dienes' steps in mathematics learning using number line media. The subjects of this research are three students out of 21 students in the 6th grade at MI Bustanul Ulum. These three students have different levels of mathematical ability: high, medium, and low. The sampling method used is purposive sampling, where the researcher selects subjects based on specific criteria relevant to the research objectives. The instruments used include observation sheets, interview guides, and a test consisting of 8 questions related to the addition and subtraction of integers. Data collection techniques refer to the Miles and Huberman model (2014), which includes data reduction, data presentation, and drawing conclusions. To ensure data validity, triangulation techniques are used, which combine data from observations, interviews, and test results. The learning steps used are the 6 steps of Dienes (1971), namely free play, rule-based play, equivalence property play, representation, symbolization, and formalization. This learning model is designed to help students understand the concept of integer operations through concrete to abstract representations. The steps can be summarized in the following diagram.

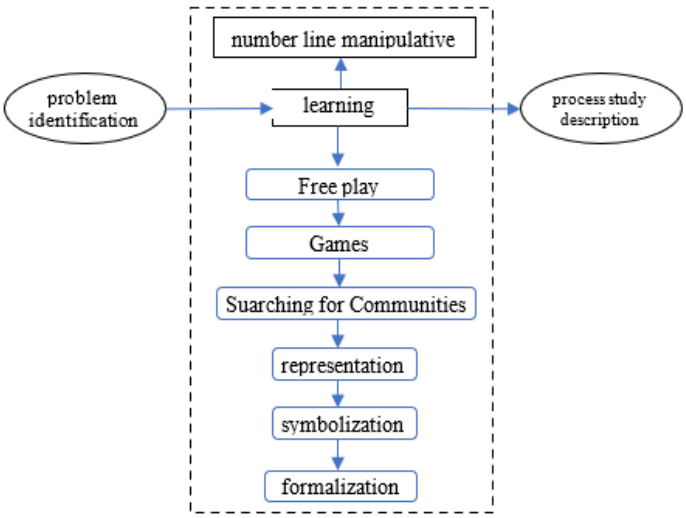

Figure 1. Research Flow Diagram

3. RESULTS AND DISCUSSION

This study begins by identifying the problems in the 6th grade of MI Bustanul Ulum. Based on observations, it was found that these 6th-grade students still face difficulties in performing operations with integers. Knowledge and understanding of integer operations are essential, as they form the foundational concepts that students need to master in order to build

further knowledge. If this foundation is weak, it will be difficult to grasp more advanced mathematical concepts. Therefore, the researcher requested three subjects with varying abilities (high, medium, and low) from the class teacher. These three subjects were then taught using the Dienes steps with a number line manipulative outside regular class hours. To assess the students' initial abilities, the researcher gave a test on integer addition and subtraction. The test consisted of 8 questions. The procedure involved students individually answering the 8 questions within 15 minutes. The results of the pre-test are shown in **Table 2.**

Table 2. Pre-test Results

No.	Names	Individual Score
1	Wan	25
2	Afa	50
3	Ais	75
	Average	50

Based on the **Table 2**, the average score of the students in understanding integer addition and subtraction operations is 50. This indicates that the students' understanding of these operations is still lacking. The next step was to apply the 6 Dienes steps using the number line manipulative. The aim of implementing these 6 steps is to help address the students' difficulties in integer addition and subtraction operations. Below is an explanation of the activities that took place during each step.

Data Reduction

Free Play: In this stage, students were provided with the number line manipulative and allowed to freely explore and play with it without strict rules. The purpose of this phase is to make students comfortable with the manipulative and start recognizing features of the number line, such as the starting point, integer points, and the spacing between numbers. During this step, students discover their own rules, such as moving to the right (for positive numbers) and moving to the left (for negative numbers).

Figure 2. Number Line Manipulative

At this stage, students, working in groups, were able to solve 4 problems using the rules they created themselves (right and left movements). The problems were: 2+3, 2-3, -2+3, and -2-3. Below is the detailed conversation that took place.

Teacher : How did you solve these problems using the Number Line?
Student Afa : By moving the figure to the right and left.
Teacher : How do you do that?
Student Afa : For example, 2+3 means starting from zero, move 2 units to the right, then move another 3 units to the right. The figure stops at number 5, so 2+3=5.
Teacher : Alright, how about the others? Did you understand it?
Students : Yes.
Teacher : So, what about -2+3, Mr. Ais?
Student Ais : Starting from zero, the figure moves 2 units to the left, then moves 3 units to the right. The figure ends up at number 1, so -2+3=1.
Teacher : Good, what about -2-3, Mr. Wan?
Student Wan : Starting from zero, the figure moves 2 units to the left, then moves 3 more units to the left. The figure ends up at -5, so -2-3=-5.

These problems were successfully solved by the 3 subjects. However, the free rules used by the students did not allow them to solve the problems 2+(-3), 2-(-3), -2+(-3), and -2-(-3). This was captured in the conversation as follows:

Teacher : Alright, now what about 2+(-3)?
Students : Everyone is silent.
Teacher : Come on, what do you think, Mr. Afa, Mr. Ais, and Mr. Wan?
Student Afa : Starting from zero, the figure moves 2 units to the right, then...?
Teacher : Then, what happens? Maybe the others can help?
Student Ais : It's plus and minus, so which way should the figure move?

This confusion arose because of the two adjacent signs, which made the students unsure whether to move the figure to the right or to the left.

Games with Rules: At this stage, the teacher had introduced the rules for using the manipulative. The rules were as follows.

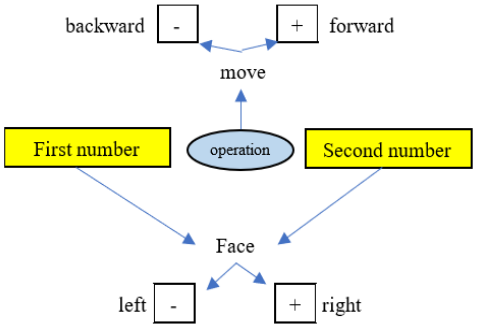

Figure 3. Rules for Integer Operations

At this stage, the students were explained the rules above using the number line manipulatives directly. Four problems that could not be solved by the students in the free play stage were successfully solved by the students using these rules. As recorded in the following conversation:

Teacher : Alright, using the rules above, how would you solve the problem 2-(-3)?
Students : (everyone raises their hand)
Teacher : Wan!
Student Wan : Starting from zero, since 2 is a positive number, the figure faces right and moves to the positive number 2. Then, because the operation is subtraction, the figure will move backward, but since the second number is also negative, the figure faces left. So, the figure faces left and moves 3 units backward. The figure stops at number 5, so 2-(-3) = 5.

By using the Facing and Moving rules, the four problems that couldn't be solved with the free play method (right and left) were successfully solved by the students.

Searching for Communities Game: In this stage, the teacher directed the students to group the results of operations that were equal. The results of the students' group work are as follows."

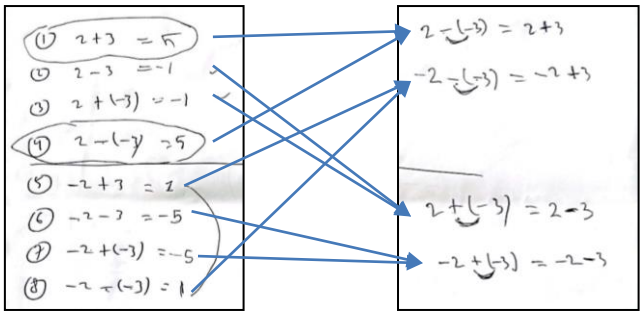

Figure 4. Grouping of Operation Results

From this grouping, the teacher prompted the students to observe the similarity of results from different operations. From this, the students discovered that the result of 2-(-3) = 2+3 and also the result of -2-(-3) = -2+3. Similarly, for the other problems, the result of 2+(-3) = 2-3 and the result of -2+(-3) = -2-3.

Representation. In this stage, the students linked the game with rules (facing and moving) to the free play game (right and left). Student Afa solved the problem 2+(-3) using the game with rules, which resulted in a series of student activities. The figure was placed at position 0, and since the first number, 2, is positive, it faced right and moved 2 units. It reached the number 2. Since the operation is positive, the figure was supposed to move forward. However, because the second number is negative, the figure faced left and moved 3 units forward, reaching the number -1. This was then simplified by student Afa using their prior knowledge. The statement 2+(-3) was transformed into 2-3. Thus, the activity became simpler: placing the figure at position 0, moving 2 units to the right to reach 2, and then moving 3 units to the left, stopping at -1.

Symbolization. In this stage, students wrote symbols by replacing the two adjacent signs with a single operation sign. Two students, Wan and Afa, still needed the number line manipulative to solve integer addition and subtraction problems. One of their works is as follows:

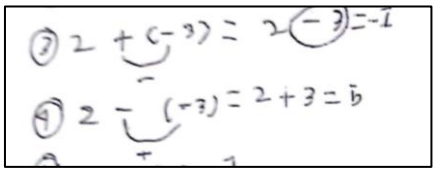

Figure 5. **Wan's Work**

In the **Figure 5**, student Wan changed the two signs (+ and -) into a single minus sign, transforming 2+(-3) into 2-3. However, after this step, student Wan still needed the number line manipulative, using the initial rule they understood. That is, placing the figure at 0, moving 2 steps to the right, and then moving 3 steps to the left. The figure stopped at -1. Therefore, it can be said that 2+(-3) = 2-3 = -1. Similarly, for the next problem, after student Wan changed the two signs (- and -) into a plus sign, 2-(-3) became 2+3. Student Wan still needed the number line manipulative, placing the figure at 0, moving 2 steps to the right, and then moving 3 steps to the right. The figure stopped at 5. Therefore, it can be said that 2-(-3) = 2+3 = 5. In contrast to student Wan, after changing the two adjacent signs into a single operation sign, Ais could directly determine the result without needing any manipulatives. From this stage, it was concluded that when there are two adjacent negative signs (not separated by numbers), they can be replaced with a positive sign. Also, when two different signs, positive and negative, are adjacent (not separated by numbers), they can be replaced with a negative sign.

Formalization. In this stage, students began to use symbols and rules formally in solving mathematical problems. They were able to solve mathematical problems with clear rules, without relying on physical manipulatives, although they still understood the meaning of the operation. Students were also able to state that if there are two adjacent signs (not separated by numbers), the two signs can be replaced with one sign: different signs become - (e.g., +- and -+), and the same signs become + (--). Below is the work of student Ais.

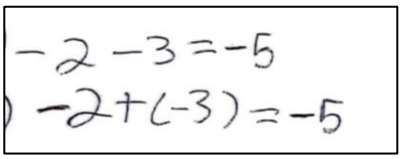

Figure 6. **Ais' Work**

Data Presentation

From the explanation above, the results can be simplified in **Table 3**.

Table 3. **Dienes' Steps and Student Activities**

No.	Dienes' Step	Student Activity
1	Free Play	In groups, students were able to solve 4 problems with rules they created themselves (right-left). These problems were:
		2+3 => move 2 steps to the right, move 3 steps to the right
		2-3 => move 2 steps to the right, move 3 steps to the left
		-2+3 => move 2 steps to the left, move 3 steps to the right
		-2-3 => move 2 steps to the left, move 3 steps to the left
		The other 4 problems could not be solved with this free play.
2	Game with Rules	With the Facing and Moving rule, the 4 problems that could not be solved with free play could now be solved with this rule. The explanation is as follows:
		2+(-3) => face right and move 2 steps, face left and move 3 steps
		2-(-3) => face right and move 2 steps, face left and move 3 steps backward
		-2+(-3) => face left and move 2 steps, face left and move 3 steps
		-2-(-3) => face left and move 2 steps, face left and move 3 steps backward
3	Searching for Communities	Students grouped results from operations that were the same with the teacher's guidance.
		The result of 2-(-3) = 2+3, which is 5
		The result of -2-(-3) = -2+3, which is 1
		The result of 2+(-3) = 2-3, which is -1
		The result of -2+(-3) = -2-3, which is -5
4	Representation	Students linked the game with rules (facing and moving) to the free play game (right-left). Students practiced with the Number Line manipulative, where the Facing and Moving rule could be replaced by the Right and Left rule as in step 3 by first simplifying the two adjacent signs into one sign.
5	Symbolization	Students wrote symbols by replacing two adjacent signs with a single operation sign.
		The problem 2-(-3) became 2+3
		The problem -2-(-3) becomes -2+3
		The problem 2+(-3) becomes 2-3
		The problem -2+(-3) becomes -2-3
6	Formalization	Students begin to use symbols and rules formally in solving mathematical problems. Students can also say that if there are two adjacent signs (not separated by numbers), these two signs can be replaced with one sign, such as:
		Different signs become – (+- and -+)
		Same signs become + (--)

After the learning activities using the number line teaching aid and applying Dienes' steps, the three students were given another set of 8 problems. These problems were completed individually within 15 minutes. Students were free to choose whether to use the teaching aid or not. The results are shown in **Table 4**.

Table 4. Final Test Results

No	Name	Score (Individual)
1	Wan	75
2	Afa	87.5
3	Ais	100
Average		**87.5**

Based ont the **Table 4**, the average score of the students in understanding the concept of addition and subtraction of integers is 87.5. This indicates an improvement in the students' understanding of integer addition and subtraction operations. This aligns with Ramla's (2024) research, which states that learning through games can improve academic achievement. Additionally, studies by Ananda (2017) and Ulfa (2023) also support this, emphasizing that learning with teaching aids can enhance students' understanding and learning outcomes. Furthermore, Sari's (2017) research revealed that applying Dienes' principles in learning has a positive impact on students' learning achievements.

4. CONCLUSION

This study aims to describe the utilization of the Number Line as a teaching aid in implementing Dienes' Steps to address students' difficulties in performing addition and subtraction operations with integers. In the **free play step**, students used their own rules, namely "right and left," to solve part of the integer operation problems. In the **games with rules step**, the teacher introduced the "face and move" rule to help students solve problems that could not be completed during the free play stage. During the **search for commonalities step**, the teacher guided students to group identical answers from integer operations. In the **representation step**, students connected the "face and move" rule with the "right and left" rule using the Number Line teaching aid. In the **symbolization step**, students simplified two adjacent signs into a single representative sign. Finally, in the **formalization step**, students concluded that two different adjacent signs could be combined into a negative sign (e.g., +- and -+ become -) and two similar adjacent signs could be combined into a positive sign (-- becomes +). The results indicate that learning through Dienes' Steps, combined with the use of a Number Line teaching aid, effectively addressed students' difficulties in performing integer operations.

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